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VIBRATIONS OF A CIRCULAR CYLINDER WHICH VARIES IN THICKNESS WITH TIME

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VIBRATIONS OF A CIRCULAR CYLINDER WHICH VARIES IN THICKNESS WITH TIME

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Abstract. Vibrations of a cylinder which a thickness varying in time have been examined in a number of earlier studies, but the authors employed different simplifying assumptions, such as assuming that the length of the cylinder is infinitely great or that axial symmetry exists. Accordingly, only a plane-strained state of the cylinder was investigated. This paper demonstrates that with a change in cylinder thickness new and interesting phenomena arise which accompany the vibration process. For example, the process of vibrations of a cylinder to its total disappearance occurs in the form of three types of waves. One of them is a direct wave, the second a reflected wave, and the third a doubly reflected wave.

The vibrations of a cylinder with a thickness varying in time have been examined in [1, 2, 4]; the authors made use of different simplifying assumptions, for example, assuming that the length of the cylinder is infinitely large or that axial symmetry exists. Accordingly, only the plane-strained state of a cylinder was investigated. It will be demonstrated below that with a change in cylinder thickness new and interesting phenomena occur which accompany the vibration process. /77*

§ 1. Formulation of Problem.

We will examine a circular cylinder of the length l, whose thickness varies in time in such a way that the outer radius of the cylinder is R and whose inner radius is $r = r_0 + vt$, where R, r_0 and v are some constants.

We will assume that a surface force acts on the inner surface of the cylinder. After matching the z-axis of the cylindrical coordinate system

*Numbers in margin indicate pagination in foreign text.

(r, θ, z) with the cylinder axis and assuming that ν, the Poisson coefficient of cylinder material is close to 0.5, we write the equations from the dynamic theory of elasticity

$$\begin{gathered}
\frac{a_1}{a_2}\frac{\partial \psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2 W}{\partial \theta^2}+\frac{\partial^2 W}{\partial z^2}-\frac{\partial^2 W}{\partial t^2}=0; \\
\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}-\frac{V}{r^2}+\frac{\partial^2 V}{\partial z^2}-\frac{\partial^2 V}{\partial t^2}=-\frac{a_1}{a_2}\frac{1}{r}\frac{\partial \psi}{\partial \theta}; \\
\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}+\frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2}-\frac{\partial^2 U}{\partial t^2}=-\frac{a_1}{a_2}\frac{\partial \psi}{\partial z}; \\
\psi=\frac{\partial W}{\partial r}+\frac{W}{r}+\frac{\partial U}{\partial z}+\frac{1}{r}\frac{\partial V}{\partial \theta}.
\end{gathered} \tag{1.1}$$

Here r, θ, z and U, V, W are the coordinates and components of movement of an arbitrary cylinder point, related to R; $t = \frac{\bar{t}}{R}\sqrt{a_2/\rho}$ is dimensionless time, and $\bar{t}$ is real dimensional time;

$$a_1=\frac{E}{1+\nu}\left(1+\frac{\nu}{1-2\nu}\right); \qquad a_2=\frac{E}{2(1+\nu)}; \qquad \psi=\frac{(1+\nu)(1-2\nu)}{\nu E}\sigma_{rr};$$

ρ, E are the density and elastic modulus of cylinder material.

Without impairing the generality of our discussion, it is assumed that /78 the surface force has the intensity

$$P = A_{mn}(t)\sin n\theta \cos \lambda_m z, \tag{1.2}$$

where $A_{mn}(t)$ is an arbitrary function of time.

If the following conditions are satisfied at the ends of the shell

$$U = \sigma_{zz} = \sigma_{rz} = 0, \tag{1.3}$$

the solution for a general case of loading of the shell can be obtained by applying the superposition principle and determining the $A_{mn}(t)$ value as the Fourier coefficient of a function representing the load. Assuming that m

ranges through a series of fractional numbers, we can also satisfy the other conditions at the ends of the shell.

Taking these conditions into account, we seek the solution in the form

$$W = \overline{W}(r, t)\sin n\theta \cos \lambda_m z; \qquad V = \overline{V}(r, t)\cos n\theta \cos \lambda_m z;$$
$$U = \overline{U}(r, t)\sin n\theta \sin \lambda_m z. \tag{1.4}$$

Substituting the U, V, W values into equation (1.1) and applying the operational method, based on a unilateral Laplace transform, to them, we obtain the following system for determining the representations of the functions $\overline{\psi}(r, t) \prec\!\div \psi(r, p)$, $\overline{U}(r, t) \prec\!\div U(r, p)$, $\overline{V}(r\ t) \prec\!\div V(r, p)$, $\overline{W}(r, t) \prec\!\div W(r,\ p)$

$$\frac{a_1}{a_2}\frac{\partial \psi}{\partial r} - \left(\frac{n^2}{r^2} + \lambda_m^2 + p^2\right) W = 0;$$
$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} - \left(\lambda_m^2 + p^2 + \frac{1}{r^2}\right) V = -\frac{a_1}{a_2}\frac{n}{r}\psi; \tag{1.5}$$
$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} - \left(p^2 + \frac{n^2}{r^2}\right) U = -\frac{a_1}{a_2}\lambda_m \psi; \qquad \frac{\partial W}{\partial r} + \frac{W}{r} = \psi + \lambda_m U + \frac{n}{r} V.$$

It can be seen that equations (1.5) will be satisfied if the U, V, W components are determined as follows:

$$V = X_1 I_1(k_1 r) + Y_1 K_1(k_1 r) -$$
$$- k_1 n \frac{a_1}{a_2}\left[I_1(k_1 r)\int_1^r K_1(k_1 r)\,\psi dr - K_1(k_1 r)\int_1^r I_1(k_1 r)\,\psi dr\right];$$
$$U = X_2 I_n(pr) + Y_2 K_n(pr) - \tag{1.6}$$
$$- p\lambda_m \frac{a_1}{a_2}\left[I_n(pr)\int_1^r rK_n(pr)\,\psi dr - K_n(pr)\int_1^r rI_n(pr)\,\psi dr\right];$$
$$W = \frac{X_3}{r} + \frac{1}{r}\int_1^r r\left(\psi + \lambda_m U + \frac{n}{r} V\right) dr.$$

Here ψ is solution of the integral equation

$$\psi(r) = f(r) + \frac{a_2}{a_1}\int_1^r M(r, \xi)\,\psi(\xi)\,d\xi + Y_3; \tag{1.7}$$

$I_1(k_1 r)$, $I_n(pr)$ and $K_1(k_1 r)$, $K_n(pr)$ are the Bessell functions of the first and second kinds respectively; X_i, Y_i are representation of some arbitrary functions of time; $k_1 = \sqrt{\lambda_m^2 + p^2}$. /79

Since the process of decrease in thickness occurs over a brief time interval, the f(r) and M(r, ξ) values must be determined for large p values. Using asymptotic representations of the Bessell functions, we write

$$M(r, \xi) = \frac{1}{\sqrt{\xi}} r^{-3/2}\,\frac{a_1}{a_2}\left(\frac{n^2}{r^2} - \lambda_m^2 \xi\right) \operatorname{sh} p(r - \xi);$$

$$f(r) = X_3 p^2 \ln r + X_1 \frac{np^2}{\sqrt{2\pi p}} \int_1^r \frac{1}{\eta}\int_1^\eta \frac{e^{p\xi}}{\sqrt{\xi}}\,d\xi d\eta +$$

$$+ X_2 \frac{\lambda_m p^2}{\sqrt{2\pi p}} \int_1^r \frac{1}{\eta}\int_1^\eta \sqrt{\xi}\, e^{p\xi} d\xi d\eta + Y_1 np^2 \sqrt{\frac{\pi}{2p}} \int_1^r \frac{1}{\eta}\int_1^\eta \frac{e^{-p\xi}}{\sqrt{\xi}}\,d\xi d\eta + \tag{1.8}$$

$$+ Y_2 \lambda_m p^2 \sqrt{\frac{\pi}{2p}} \int_1^n \frac{1}{\eta}\int_1^r \sqrt{\xi}\, e^{-p\xi} d\xi d\eta.$$

After satisfying the boundary conditions on the outer surface of the shell, which essentially involves the requirement that the components of the stress tensor be equal to zero when r = 1, we find

$$Y_1 = X_1 \frac{e^{2p}}{\pi} + X_3 \sqrt{\frac{2}{\pi p}}\sqrt{r} e^{pr}; \quad Y_2 = X_2 \frac{e^{2p}}{\pi} + X_3 \sqrt{\frac{2}{\pi p}}\sqrt{r} e^{pr}; \tag{1.9}$$

$$Y_3 = 0.$$

If we ignore the term $1/p^2$ in comparison with unity, solution of equation (1.7) is obtained in the form

$$\psi = \overline{X}_1 \psi_1 + \overline{X}_2 \psi_2 + \overline{X}_3 \psi_3, \tag{1.10}$$

where

$$\psi_1 = \psi_4 n \left\{ \frac{a_2}{a_1} \left[\frac{e^{pr} + e^{p(2-r)}}{r^{3/2}} \left(1 - \frac{\ln r}{2} \right) - 2e^p \right] - \right.$$
$$\left. - \frac{4e^p}{p^2 r^{3/2}} \left[\left(\frac{n^2}{\sqrt{r}} - \lambda_m^2 \right) \operatorname{sh} p(1-r) + \lambda_m^2 \operatorname{ch} p(1-r) \right] \right\} \frac{1}{p^2 e^p};$$
$$\psi_2 = \psi_4 \frac{\lambda_m}{p^2 e^p} \frac{a_2}{a_1} \left[e^p (\ln r - 2) + \frac{e^{pr} + e^{p(2-r)}}{\sqrt{r}} \right]; \qquad (1.11)$$
$$\psi_3 = \psi_4 \cdot \frac{1}{p^3} \left\{ \left[p^2 \ln r + \frac{n^2 e^{p(1-r)}}{p r^{3/2}} (1 - 0{,}5 \ln r) - \frac{n^2}{p} + \frac{\lambda_m^2 e^{p(1-r)}}{p\sqrt{r}} - \right. \right.$$
$$\left. \left. - \frac{\lambda_m^2}{p} \right] \frac{a_2}{a_1} - r^{-\frac{3}{2}} \left[\left(\frac{n^2}{\sqrt{r}} - \lambda_m^2 \right) \operatorname{sh} p(1-r) + \lambda_m^2 \operatorname{ch} p(1-r) \right] \right\};$$
$$\psi_4 = \frac{1}{1 + \frac{1}{p r^{5/2}} \left(\frac{n^2}{\sqrt{r}} - \lambda_m^2 \right)}; \qquad \overline{X}_i = \frac{p^2 e^p}{\sqrt{2\pi p}} X_i \quad (i = 1, 2); \quad \overline{X}_3 = p^3 X_3,$$

which is easy to confirm by substituting the ψ value, determined using formula (1.10) into the initial equation. The problem is to find $\overline{X}_i$, which satisfies the boundary conditions on the moving surface. After determining $\sigma_{rr}(r, p)$, $\sigma_{r\theta}(r, p)$, $\sigma_{rz}(r, p)$, with $r = r_0 + vt$, we write the boundary conditions in the following form

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$$\sigma_{rr}(r, p) \dashrightarrow \sigma_{rr}(r, t) = -P(t);$$
$$\sigma_{r\theta}(r, p) \dashrightarrow \sigma_{r\theta}(r, t) = 0; \qquad \sigma_{rz}(r, p) \dashrightarrow \sigma_{rz}(r, t) = 0, \qquad (1.12)$$

here v is the rate of decrease in wall thickness, related to the propagation velocity of the distortion wave which henceforth will be called the critical velocity.

§ 2.

We introduce the following notations:

(Formula 2.1 on following page)

$$L_1 = r_0 + vt; \qquad L_2 = 1 - \ln L_1; \qquad L_3 = 1 - 0{,}5 \ln L_1;$$

$$N_i(t) = \int_{vt}^{t} X_i(\tau)\, d\tau; \qquad F_i(t) = \int_{2(1-r_0)-vt}^{t} X_i(\tau)\, d\tau \qquad (i = 1, 2);$$

$$N_3(t) = \frac{n^3 a_1}{2a_2 L_1^3} \int_{vt}^{t} X_3(\tau)(\tau - vt)\, d\tau;$$

$$F_3(t) = \frac{L_1^{-\frac{3}{2}}}{2}\left(n^2 L_1^{-\frac{1}{2}} - 2\lambda_m^2\right)\frac{a_1}{a_2} \int_{2(1-r_0)-vt}^{t} X_3(\tau)(\tau - 2 + 2r_0 + vt)\, d\tau;$$

$$\varepsilon_1(t) = a_{11}\left[-vtN_1 + \int_{vt}^{t} X_1(\tau)\,\tau d\tau\right] + a_{12}\left[-vtN_2 + \int_{vt}^{t} X_2(\tau)\,\tau d\tau\right] + a_{13}N_3;$$

$$\varepsilon_2(t) = a_{21}N_1 + a_{22}N_2 + a_{23}N_3;$$

$$\varepsilon_3(t) = a_{31}N_1 + a_{32}\left[-vtN_2 + \int_{vt}^{t} X_2(\tau)\,\tau d\tau\right] + N_3; \qquad (2.1)$$

$$Q_1(t) = a_{11}\left\{[-2(1-r_0) + vt]F_1 + \int_{2(1-r_0)-vt}^{t} X_1(\tau)\,\tau d\tau\right\} +$$

$$+ a_{12}\left\{[vt - 2(1-r_0)]F_2 + \int_{2(1-r_0)-vt}^{t} X_2(\tau)\,\tau d\tau\right\} + a_{13}F_3;$$

$$Q_2(t) = a_{21}F_1 + a_{22}F_2 + a_{23}F_3;$$

$$Q_3(t) = -a_{31}F_1 + F_3; \qquad a_{11} = nL_1^{-\frac{3}{2}} \cdot L_3;$$

$$a_{12} = \lambda_m L_1^{-\frac{1}{2}}; \qquad a_{13} = -\frac{a_2}{a_1}\frac{L_1}{n}; \qquad a_{21} = \lambda_m a_{11};$$

$$a_{22} = L_1^{-\frac{1}{2}}; \qquad a_{23} = -\frac{L_1}{n}; \qquad a_{31} = -a_{22}; \qquad a_{32} = n\lambda_m L_1^{-\frac{3}{2}};$$

$$M_{js}^{*} = \int_{1-r_0}^{t} X_j(\tau)(\tau - 1 + r_0)^s\, d\tau.$$

Now, for determining $X_i(t) \leftarrow\!\!\div \overline{X}_i(p)$ we write a system of three integral equations:

$$\varepsilon_1(t) = -\frac{a_1}{a_2}P(t); \qquad \varepsilon_2(t) = 0; \qquad \varepsilon_3(t) = 0; \qquad (2.2)$$

$$\text{for } 1 - r_0 \leqslant t \leqslant \frac{2(1-r_0)}{1+v}$$

$$\varepsilon_1(t) - 2nM_{11} + \lambda_m(\ln L_1 - 2)M_{21} + \ln L_1 \cdot M_{30} = T_1(t) = -\frac{a_1}{a_2}P(t);$$

$$\varepsilon_2(t) + \frac{\lambda_m}{L_1}M_{30} = T_2(t) = 0; \qquad \varepsilon_3(t) + \frac{nL_2}{L_1^2}M_{30} = T_3(t) = 0; \tag{2.3}$$

for $\dfrac{2(1-r_0)}{1+v} \leqslant t \leqslant \dfrac{1-r_0}{v}$

$$T_1(t) + Q_1(t) = -\frac{a_1}{a_2}P(t); \qquad T_2(t) + Q_2(t) = 0; \qquad T_3(t) + Q_3(t) = 0. \tag{2.4}$$

We note that with acceptable accuracy the system of equations (2.2) can be written in the following, approximately equivalent form:

$$\int_{vt}^{t} X_i(\tau)\,d\tau = \frac{A_i(t,v)}{B(t,v)}P(t), \tag{2.5}$$

where $A_1 = -(1+\lambda_m)L_1^{-\frac{1}{2}}; \qquad A_2 = \lambda_m n L_1^{-\frac{3}{2}}L_2 - \dfrac{\sqrt{L_1}}{n}; \qquad A_3 = \lambda_m^2 n^2 L_1^{-3}L_2;$

$$B = \frac{t(1-v)}{2}\left\{\left[\frac{\lambda_m}{n} + \frac{a_2}{a_1}\frac{2}{t(1-v)}\right] + \frac{(1+\lambda_m)n}{L_1}L_3 - \frac{\lambda_m^2 n(a_1+a_2)}{a_1 L_1}L_2\right\}.$$

It is easy to write a solution of equations (2.5) in particular, if $A_{mn} = A_0 \sin\omega t$, it assumes the form

$$X_i(t) = A_0\left[\omega\sum_{n=0}^{\infty} v^n \frac{A_i(z)}{B(z)}\cos\omega v^n t + \right.$$

$$\left. + \sum_{n=0}^{\infty}\frac{\dot{A}_i(z)B(z) - A_i(z)\dot{B}(z)}{B^2(z)}v^{n+1}\sin\omega v^n t\right]. \tag{2.6}$$

Here $\dot{A}_i$ and $\dot{B}$ are derivatives of z; $z = v^{n+1}t$.

Since $v \ll 1$, the series (2.6) converge rapidly. /82

Equations (2.3) also can be written in the form (2.5). This requires that values of the second order of magnitude be neglected. Accordingly, their solution can be obtained by analogy, taking (2.6) into account, specifically

$$X_i(t) = A_0\left[v\omega\sum_{n=0}^{\infty}v^n\frac{A_i(y)}{B(y)}\cos\omega z + \omega\sum_{n=0}^{\infty}v^n\frac{A_i(z)}{B(z)}\cos\omega v^n t + \right.$$
$$+\sum_{n=0}^{\infty}v^{n+1}\frac{\dot{A}_i(y)B(y) - A_i(y)\dot{B}(y)}{B^2(y)}\sin\omega z + \qquad (2.7)$$
$$\left.+\sum_{n=0}^{\infty}v^{n+1}\frac{\dot{A}_i(z)B(z) - A_i(z)\dot{B}(z)}{B^2(z)}\sin\omega v^n t\right],$$

where

$$y = v^{n+2}t.$$

Using a similar approach, we find the solution of (2.4). Denoting the solution of equations (2.2) and (2.3) by $X_i^{(1)}$ and $X_i^{(2)}$, we assume

$$\int_{vt}^{2(1-r_0)-vt}X_i^{(2)}(\tau)\,d\tau + \int_0^{vt}X_i^{(1)}(\tau)\,d\tau = Y_{ji}; \qquad \int_0^{vt}X_j^{(s)}(\tau)(\tau - vt)\,d\tau = P_j^{(s)};$$
$$\frac{n^2L_1^{-\frac{1}{2}} - 2\lambda_m^2}{2L_1^{1/2}}\left\{\int_0^{vt}X_3^{(1)}(\tau)[\tau - 2(1-r_0) + vt]\,d\tau + \right.$$
$$\left.+\int_{vt}^{2(1-r_0)-vt}X_3^{(2)}(\tau)[\tau - 2(1-r_0) + vt]\,dt\right\} = R;$$
$$2\int_0^{vt}X_j^{(1)}(\tau)[\tau - 1 + r_0]\,d\tau + \int_{vt}^{2(1-r_0)-vt}X_i^{(2)}(\tau)[\tau - 2(1-r_0) + vt]\,d\tau = Z_j;$$
$$b_1(t) = \frac{L_1}{n}\left(\frac{\lambda_m^2L_2 - L_3}{\lambda_mL_3}\right) + \frac{a_1\lambda_mL_1L_2}{a_2nL_3} - \frac{L_1}{n}(t - 1 + r_0);$$
$$b_2(t) = \frac{v}{nL_3}\left[\left(1 + \frac{a_1}{a_2}\right)\lambda_mL_2 + \frac{0{,}5}{\lambda_m} - \lambda_m - \frac{L_3}{\lambda_m}\right] +$$
$$+\frac{v}{nL_3}\left[0{,}5\lambda_m\left(1 + \frac{a_1}{a_2}\right)L_2 - L_3\left(\frac{0{,}5}{\lambda_m} + \frac{a_1}{a_2}\lambda_m\right) - \frac{v}{n}(t - 1 + r_0) - \frac{L_1}{n}\right];$$

(Formulas continued on next page)

$$b_3(t) = \frac{L_1}{n\lambda_m}\left(\frac{L_3 - \lambda_m^2 L_2}{L_3}\right)\left(-\frac{L_1}{n}Y_{12} - \frac{n^3}{2L_1^2}P_3^{(1)} - R\right) +$$

$$+ \frac{a_1\lambda_m L_2}{a_2 L_3}\left(A_0\cos\omega t + \frac{a_2}{a_1}\,nL_1^{-\frac{3}{2}}L_3 Z_1 + \frac{a_2\lambda_m L_1^{-\frac{1}{2}}}{a_1}Z_2 - \frac{n^2}{2L_1^2}P_3^{(1)} - L_1^{-2}R\right);$$

$$b_4(t) = \frac{a_2 nL_1^{-\frac{3}{2}}L_3}{a_1}Y_{11} + \lambda_m nL_1^{-\frac{3}{2}}L_2 Y_{22} - \frac{n^2}{2L_1^2}P_3^{(1)} - L_1^{-2}R;$$

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$$b_5(t) = \dot{b}_3 - (t - 1 + r_0)\dot{b}_4; \qquad b_6 = b_2 + \frac{L_1}{n}; \qquad C_{21} = -\frac{\lambda_m nL_1^{-\frac{3}{2}}}{2}(2+3L_2);$$

$$C_{22} = -0{,}5vL_1^{-\frac{3}{2}}; \qquad B_1 = \frac{b_6}{b_1}; \qquad B_2 = \frac{b_5}{b_1}; \qquad \varepsilon_{21} = \frac{2C_{21}}{b_1};$$

$$\varepsilon_{22} = \frac{2C_{22}}{b_1}; \qquad T_{12}(t) = -\frac{L_1^2 0{,}5}{n^2\lambda_m^2(t - 1 + r_0)}Y_{12} -$$

$$-\frac{0{,}5n^2 L_1^{-\frac{3}{2}}}{2\lambda_m(t - 1 + r_0)}P_3^{(1)} - \frac{L_1^{\frac{1}{2}}R}{2(t - 1 + r_0)n\lambda_m^2};$$

$$T_{22}(t) = \frac{0{,}5L_1^{\frac{3}{2}}Q(t)}{n\lambda_m(t - 1 + r_0)}; \qquad Q(t) = e^{-\int_0^t B(x)dx};$$

$$T_{11}(t) = \frac{a_1 L_1^{3/2}}{2(t - 1 + r_0)a_2 nL_3}\left[A_0\sin\omega t + \frac{a_2}{a_1}\frac{nL_3}{L_1^{3/2}}Z_1 + \frac{a_2}{a_1}\frac{\lambda_m}{L_1^{1/2}}Z_2 +\right.$$

$$\left. + L_1^{-\frac{1}{2}}R\left(1 - \frac{a_2}{a_1}\right) - \frac{a_2}{a_1}\frac{L_1^2}{n^2}Y_{12}\right]; \qquad T_{21}(t) = -\frac{(a_1 + a_2)L_1}{2a_1 n(t - 1 + r_0)}Q(t);$$

$$T_{31}(t) = B_2 + (t - 1 + r_0)\left\{\sum_{i=1}^{2}\varepsilon_{2i}\left[\int_0^t \frac{T_{1i}(\tau)}{(\tau - 1 + r_0)}d\tau +\right.\right.$$

$$\left.\left. + \int_0^t \frac{T_{2i}(\tau)}{(\tau - 1 + r_0)}\int_0^\tau \frac{B_2(x)}{Q(x)}dxd\tau\right]\right\};$$

$$T_{41}(t) = B_2 + (t - 1 + r_0)\left\{\sum_{i=1}^{2}\varepsilon_{2i}\left[\int_0^t \frac{T_{1i}(\tau)}{(\tau - 1 + r_0)}d\tau +\right.\right.$$

$$\left.\left. + \int_0^t \frac{T_{2i}(\tau)}{(\tau - 1 + r_0)}\int_0^\tau \frac{T_{2i}(x)}{Q(x)}dxd\tau\right]\right\};$$

$$\varphi_i = T_{2i}\int_0^t \frac{T_{41}(\tau)}{Q(\tau)}d\tau + T_{1i} \qquad (i = 1, 2).$$

Now the solution of (2.4) is written in the form

$$X_i(t) = \dot{\varphi}_t + \frac{1}{t-1+r_0}\varphi_t \qquad (i = 1, 2);$$

$$X_3(t) = \frac{1}{2(t-1+r_0)}\left[-T_{41}(t) + B_1(t)\,Q(t)\int_0^t T_{41}(\tau)\,Q^{-1}(\tau)\,d\tau\right].$$

The v^4 value is neglected in comparison with unity.

Substituting the determined X_i and Y_i values into (1.6), and proceeding to the original, we determine W(r, t), V(r, t), U(r, t). For example, /84

for $0 \leqslant t \leqslant 1-r_0$

$$W(r,t) = W^{(1)} = \int_{r-r_0}^{t} (\tau - r + r_0)^2 \left\{-\frac{r^{-\frac{3}{2}}}{4}(1 - 0{,}5\ln r)\,X_1(\tau) - X_2(\tau) + \right.$$

$$\left. + \frac{a_1 n^2}{a_2 8 r^2} X_3(\tau) - [n^2 r^{-\frac{3}{2}}(1 - 0{,}5\ln r) - \lambda_m^2 r^{-\frac{1}{2}}]\,X_3(\tau)\right\} d\tau;$$

for $1-r_0 \leqslant t \leqslant 2-r-r_0$

$$W(r,t) = W^{(2)} = W^{(1)} + \int_{1-r_0}^{t} (\tau - 1 + r_0)^3 \left[\frac{r^{-5/2}}{6}(1-0{,}5\ln r)X_2(\tau) + \frac{X_3(\tau)}{r}\right] d\tau;$$

for $2-r-r_0 \leqslant t$

$$W(r,t) = W^{(1)} + W^{(2)} + \int_{2-r+r_0}^{t} (\tau - 2 + r + r_0)^2 \left[\frac{r^{-\frac{3}{2}}}{4}(1 - 0{,}5\ln r)\,X_1(\tau) + \right.$$

$$\left. + \frac{r^{-\frac{1}{2}}}{4} X_2(\tau) - \frac{a_1 r^{-\frac{3}{2}}}{8a_2}(n^2 r^{-\frac{1}{2}} - 2\lambda_m^2)\,X_3(\tau)\right] d\tau.$$

Analyzing the solution, we arrive at the following conclusion.

1. The process of vibrations of a cylinder to its total disappearance occurs in the form of three types of waves. One of them is a direct wave, the second a reflected wave, and the third a twice-reflected wave.

2. As indicated by formulas (2.1) and (2.2), when $v \to v_{cr}$, $v_{cr} = \sqrt{a_2/\rho}$ X_1 and X_2 tend to infinity. Vibrations of a cylinder with v values close to

v_{cr} are accompanied by the propagation of only a direct wave. The strained region of the cylinder is in this case localized near the site of application of an external force, that is, near the inner surface of the cylinder. Finally, there is a marked increase in some components ($\sigma_{\theta\theta}$) of the stress tensor.

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